BULGARIAN GEOLOGICAL SOCIETY, Annual Scientific Conference "Geology 2004" 16 – 17.12.2004

MULTISCALING MODELS IN GEOSTATISTICS

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The fundamental problem of geostatistical processing is determination of the closeness or difference in behavior of variability in different object scales. The published materials during the last ten years demonstrate that the behavior of variability in different dimensional space scales is similar. In mathematical-statistical aspect this convergence is defined as fractal formalism or fractal statistics. In order to determine fractal nature it is possible to use variogram property. According to Mandelbrot (B. Mandelbrot, 1982), who has introduced fractal statistics in scientific studies, fractal estimation or Hausdorff measure, it is apart its way of deriving from variogram, has local nature. Natural is a problem, what is by a way of deriving of a global estimation of variability. Whether is the possible share dependence between two types of estimations?

Hurst, (1957) has experimentally established that the dependence between the maximal range of a given dataset from a given experimental population of a time series, and the standard deviation has constant value. This fundamental result evolved with the concept that the subpopulation, obtained from the times series, the so called Hurst's constants *(H)*, can show the correlation length of the process, which if with major length of dependence, available dependences of a type: **Long Range Dependence - LRD** and if short length of dependence - **Short Range Dependence - SRD**. The range of values $H \in (0,5 \div 1,0)$ put variability from long range of dependence, and at values $H \in (0,1 \div 0,5)$ put variability from short length of dependence.

The connection between fractal dimension D and Hurst coefficient H is define a proportion:

$$D + H = n + 1, \qquad (1)$$

Where n - fundamental dimension of data space examination.

Apparently, that using in geostatistics variogram analysis is the eligible instrument both for definition of fractal dimension, and for definition on Hurst coefficient. The common model is following:

$$log\gamma(d) = constant + \alpha log d + error \quad as \quad d \to 0$$

Where d is the interval between samples. Actuality α is slope of straight line in log-log scale. Agterberg (Agterberg, 1982) analyzed the thirty foot sand contours of the Lloyd Minster Sparky Oil Pool in Alberta, Canada, by means of R. Mandelbrot plot, a found a fractal dimension $D = 1,3\ (i.e.\ H = 2 - D = 0,7)$. He mentioned that the conventional techniques of geostatistics can not be applied for the interpolation and extrapolation of such fractally distributed properties. We suggested two methods for interpolating and extrapolating fractal function of space. The first method is a "smooth" one, based on kriging, and second method is a stochastic method for interpolating using fractals. For interpolation or restricted extrapolation Matheron, (Matheron, 1971), Journel, A. G. and Huijbregts (Journel, A. G. and Huijbregts, C. 1978) recommend, that most good and unbiased estimators probably will receive through a computational routine in a field $Z(x)$ is referred to measured of values $Z(x_i)$ is sought for the form:.

$$Z^*(x) = \sum_{i=1}^{n} \lambda_i Z(x_i),$$

Where the weights λ_i are calculated to ensure that the estimator is unbiased and the estimation error variance is minimum. This gives to the system of equations

$$\sum_{j=1}^{l} \lambda_j \gamma(x_i, x_j) + \mu = \gamma(x_i, x); \qquad (i = 1,2,...,n) \text{ - } "A"$$

$\sum_{j=1}^{l} \lambda_j = 1$, where λ is a Lagrange parameter, and γ is semivariogram, which one determinate as:

$$\gamma(x_i, x_j) = \frac{1}{2}\{| Z(x_i) - Z(X_j)|^2\}$$

Let we shall suspect, that $Z(x_i)$ is fractional Brownian motion with an exponential *H*, which one only know at the two ends of interval – [0, L]. Substitute in equation $"A"$ we obtain:

$$\lambda_2 \gamma(0, L) + \mu = \gamma(0, x)$$
$$\lambda_1 \gamma(0, L) + \mu = \gamma(L, x)$$
$$\lambda_1 + \lambda_2 = 1$$

Assuming that $Z(0) = 0$ definitely we obtain:

$\gamma(x_i, x_j) = \frac{1}{2}\sigma^2 | x_i - x_j |^{2H}$, as this expression can be easily solved the extrapolation interpolation formula becomes:

$$Z^*(x) = Z(L)\{\frac{1}{2}[1 - | 1 - \xi |^{2H} + \xi^{2H}]\} = Z(L)Q(\xi) \qquad (2)$$

Where $\xi = x / L$, and $Q(\xi)$ depends from independent variable and value of Hurst's constants *(H)*.

Result of computational experiments. For comparisons between the model and experimentally obtained values of H we use collection of exploration data from a copper and porphyry – copper deposits of Bulgaria - Bourgas ore region. The modern statistics suggested, that matching between the theories and experimental data, is just to demonstrate in the graph are called Q - Q plot, see Fig. 1. In the figure demonstrated that the rate of the compliance between the theory and experimental materials is rather high. It indicated that the ore mineralization in those fields has to a fractal nature. Fractal nature expressed in two possible aspects. The first aspect closed about to so-called effect of "self-similarity", which one allows will plan a likeness between different levels of analysis of variability. Existence of "self-similarity" in investigated objects, defines the multi scaling possibilities in examine variability. In practical aspect, this possibility converges with multiply result of analysis of variability that small-scale effects reproduced to large scale effects.

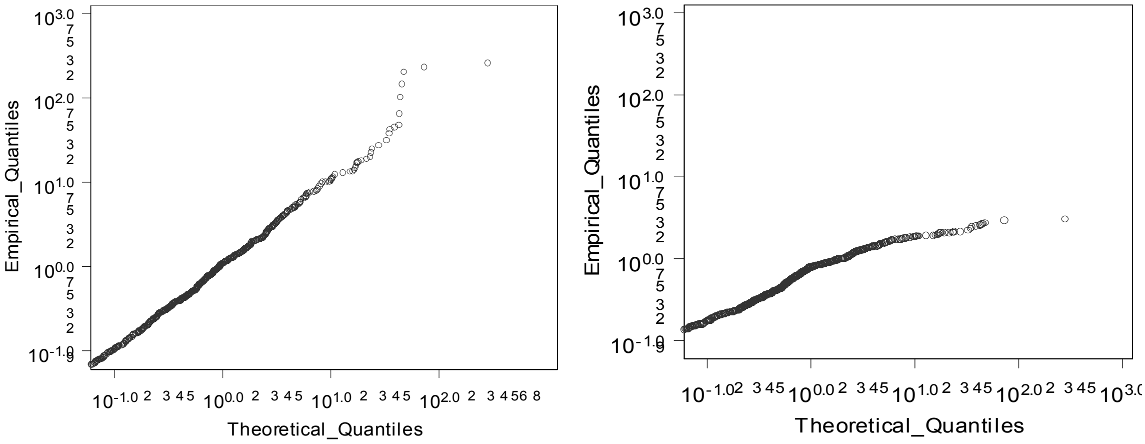

Fig. 1. In this two figures is demonstrated that the variance between standard (left) and fractal (right) Kriging is very difference

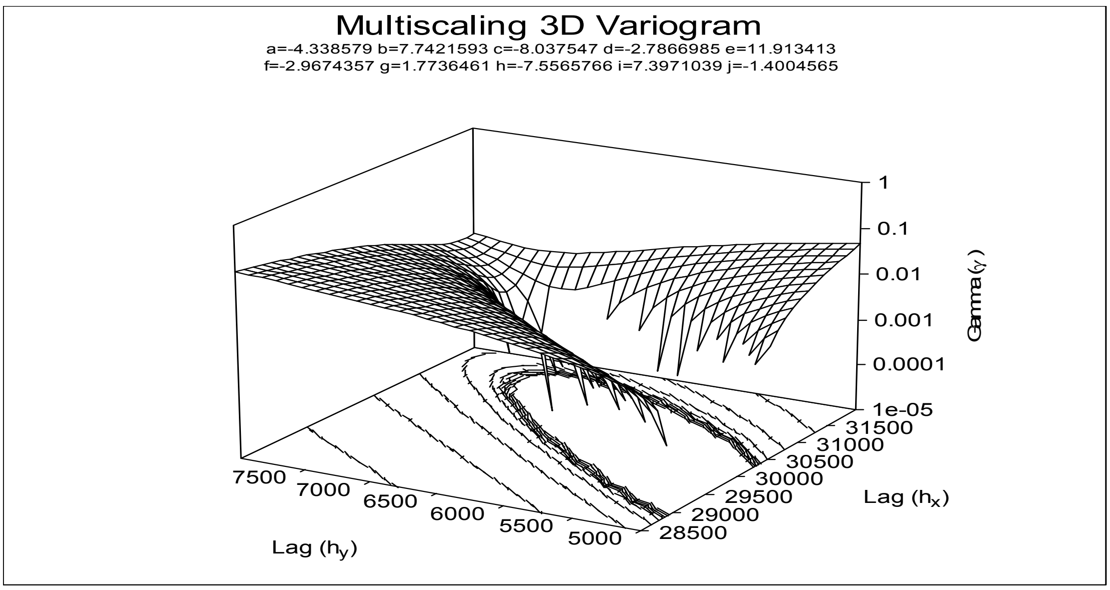

Fig. 2. Multiscaling 3D variogram based on data of Copper-Gold ore deposit "Chelopech". In top of figure the arguments of the approximating function are showed. Wide scale dependence is fixed from a "rounded" part of the function, and small-scale is fixed in bottom the profiles, where the level lines, have merged. Variances in meters is from some hundreds meter for LRD and some meters for SRD-ed.

On Fig. 2 drawing is obtained from more than 26 000 data from "Chelopech" copper and gold ore deposit. Three-dimensional variogram is obtained through a standard routine of the package Math Lab with introduced formalism (formula 2). The result of variogram analyses determined fractal dimension of investigated object. The value of fractal dimension is $D = 2,34$. In appropriate way for a coefficient on Hurst we have:

$$D + H = n + 1 \rightarrow H = 3 - 2,34 \rightarrow H = 0,66$$

It values of Hurst coefficient define the process from Long length on dependence - $\boldsymbol{H}_{\mathbf{LRD}}$). Simultaneously, from figures is visible a small scale effect available so called - Short Range Dependence – $\boldsymbol{H}_{\mathbf{SRD}}$. By figures value on $\boldsymbol{H}_{\mathbf{SRD}}$ is *0,27*. Applicable value for fractal dimension is $D = 2{,}73$. Therefore in the field are observed two levels of variability scale. The first level has long dimensional length of influencing, which one is commensurable with extension of fields. This aspect of variability, in the practical schedule is arrested by a uniform extent of mineralization, which one is inflected on a small scale and practically is fixed for total economically favorable bulk of a field. The second level is the order with one smaller a size on tree-dimensional lengths of dependence. Is probability, which this level of variability is interlinked to forming of an ore channel, which one is near to a central part of explored volume of ore deposit? Created 3D geostatistical model confirmed with an already introduced hypothesis (see Sv. Bakardjiev, 2004).

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