



A stochastic model for simulation of karst spring discharge: Case study – Seljašnica karst spring (SW Serbia)

Стохастичен модел за симулация на дебит на карстов извор на примера на карстов извор Селяшница (ЮЗ Сърбия)

Veljko Marinović, Branislav Petrović, Zoran Stevanović
Велко Маринович, Бранислав Петрович, Зоран Стеванович

Centre for Karst Hydrogeology, Department of Hydrogeology, Faculty of Mining and Geology, University of Belgrade 11000, Serbia; E-mail: veljko.marinovic@yahoo.com

Keywords: karst spring, stochastic modeling, ARCR model, Serbia.

Introduction

Karst rocks cover a significant part of the Earth's surface. Considering that karst can accumulate large amount of groundwater of excellent quality, karst aquifers are one of the most significant sources of drinking water. It is estimated that between 20% and 25% of the world's population uses karst groundwater. Among them is the population in Serbia as well, considering that almost 20% of total water supply in Serbia is from karst groundwater and it is estimated that Serbia has about $580 \times 10^6 \text{ m}^3$ of karst groundwater as potential exploitation reserves (Stevanović, 1995). Even though it seems that this amount of water can meet all water demands, the most common problem with karst groundwater is its seasonal fluctuation of quality and quantity in the course of a hydrological year. For instance, heavy rains and floods can deteriorate the quality of karst groundwater (especially its turbidity and microbiology), while seasonal depletion of karst groundwater reserves can jeopardise the local water supply. Thus, it will be very useful if one can predict the behaviour of karst groundwater quality and quantity regime, no matter if it is short- or long-term prediction. Regarding groundwater quantity, the most important thing is to predict the extreme discharge rates – minimal and maximal values, which is particularly significant in recession period when the water demand are highest and spring discharge is in minimum. Unlike intergranular aquifers, where the creation of hydrodynamic models within certain software packages is common, this is not (yet) the case with karst aquifers. Taking into account the het-

erogeneity and anisotropy of karst rocks (i.e., limestone in this case), the applicability of deterministic models is minimized, since it is practically impossible to predict the spatial distribution of karst groundwater within the karst hydrogeological system. For this reason, stochastic models in some cases may be the better option for simulation of karst spring discharge rates rather than deterministic models. Stochastic modeling of karst spring discharge has been applied to the Seljašnica karst hydrogeological system. This karst system is part of the Babine karst plateau that belongs to the Dinarides. This karst plateau (and aquifer system) is located in SW Serbia at the very border with Montenegro (probably having a transboundary character) and is drained by three large karst springs: Seljašnica and Bučje in Serbia and Breznica in Montenegro. The plateau is a fluvial relict formed over the thick deposits of Triassic limestones, whose thickness varies from 150 to 750 m, exceptionally up to 1200 m, as a bedrock of the ophiolites of Jurassic age. Numerous karst features can be found, such as sinkholes, dry and blind valleys that are proof about the existence of the formerly well-developed river network. Some ponors and small caves exist as well. The Seljašnica karst aquifer was formed in the carbonate rocks of Triassic age. The recharge process of the karst aquifer is primarily dependent on precipitation regime (effective rainfalls and melted snow). In bare karst areas, the karst aquifer is being recharged by the direct infiltration of atmospheric waters, while the aquifer can be recharged by the sinking of surface streams formed on ophiolites and young volcanic rocks (allogenic recharge) (Marinović, Petrović, 2018).

Methodology

As stated above, the ability to predict karst spring discharge rates can be very useful particularly in the recession period of the year (i.e. in summer months). This can be achieved by applying stochastic or numerical modelling techniques. Considering unpredictability of karst hydrogeological system functioning, stochastic modelling is easier to apply while at the same time the results are relatively reliable. Stochastic modelling techniques that were applied in the case of Seljašnica karst spring are autocorrelation function (ACF), cross-correlation function (CCF) and autoregressive-crossregression model (ARCR). This modelling techniques were used to simulate karst spring discharge and calculate a short-term prediction of discharge rates with an appropriate confidence level. Autocorrelation is the correlation between successive values of the same variable (Kresic, 2007), which defines a measure of the internal correlation within a time series. Cross-correlation function determines the interdependence of two random variables, a dependent and an independent one. Cross-correlation analysis gives better insight into the functioning of the karst hydrogeological system, i.e. on the discharge regime and the influence of the rainfall on the system. Autoregression (AR), crossregression (CR) and combined autoregressive-cross-regression model (ARCR) models were applied to simulate the discharge of the Seljašnica karst spring. Autoregressive models are used to generate synthetic arrays of independent time series, and in some cases, they can be used to produce short-term (1–2 days) predictions (Krešić, 1991). Besides the AR model, cross-regression (CR) models can also be used for simulation, where, besides the dependent variable (spring discharge), the independently variable (precipitation) for 1, 2, ..., n the previous days is introduced. For simulation and short-term forecasts of karst groundwater discharge, the best results are obtained by the combined (ARCR) model, which includes the AR and CR model. ARCR models belong to the category of multivariate time series models (Krešić, 2010) and include multiple linear regression. In order to improve the reliability of the model, precipitation data were transformed by using a linear moving average filter (LMAF), since the simple ARCR model includes only gross precipitation, which particularly has an impact on the spring discharge in the recession period.

Results

Stochastic modeling techniques have been applied on time series of discharge and rainfall data. In the case of Seljašnica karst spring, time series of daily values of spring discharge as well as daily

precipitation values from the Sjenica rain gauge in the period May 2016–May 2018 were considered. Autocorrelogram gives a preliminary insight into the characteristics of the karst hydrogeological system, i.e. the spring discharge characteristics and its behavior during a defined time series. The autocorrelation of the spring discharge data shows that the system memory is about 40 days, after which the random variable becomes independent. However, peaks can be observed between days 40 and 58, as well as after day 85, which can be triggered by snowmelt and infiltration. On the other hand, autocorrelation of precipitation data shows a system memory of only 2 days, which is understandable taking into account the random nature of this variable. The cross-correlogram shows the reaction of large karst conduits with a delay of two days, while the statistical influence of precipitation on the discharge of Seljašnica karst springs exists until the 25th day. Also, peaks are observed around 18th day, which could generally be taken as the reaction time of the entire catchment area to the precipitation, or to the general porosity of this aquifer. As for application of the AR and CR models, the coefficients of the autoregressive model range from 0.922 for the model order 1 to 0.937 for the model order 10 which is the consequence of the long-term memory. In contrast, the coefficients of the cross-regression model are low and range from 0.03 for the model order 1 to 0.36 for model order 10. The coefficients of the combined autoregression-cross-regression (ARCR) model vary in the range from 0.928311 for model order 1 to 0.942365 for the model order 10. It can be seen that with increasing order of the model, the validity of the model does not increase significantly, based on the coefficients of the model. Therefore, Krešić (2010) states that when the time series are highly autocorrelated, the order of model 1 or 2 (exceptionally up to 4) gives quite satisfactory results in hydrogeological practice, which was used for the simulation of the Seljašnica spring discharge (Fig. 1). Fig. 1 shows that high and average water periods are generally well simulated, while the highest oscillations occur during low water periods. Such a result is due to the fact that this model takes gross precipitation, which affects the large differences in measured and simulated values during the recession period. A linear moving average filter was applied to avoid these problems. This filter transforms the gross precipitation based on a predetermined moving average window, which is expressed in days. A moving average window of 40 days was applied to the existing ARCR model, which generally corresponds to the memory of the karst hydrogeological system calculated using the autocorrelation function. Thus, Figure 1 clearly

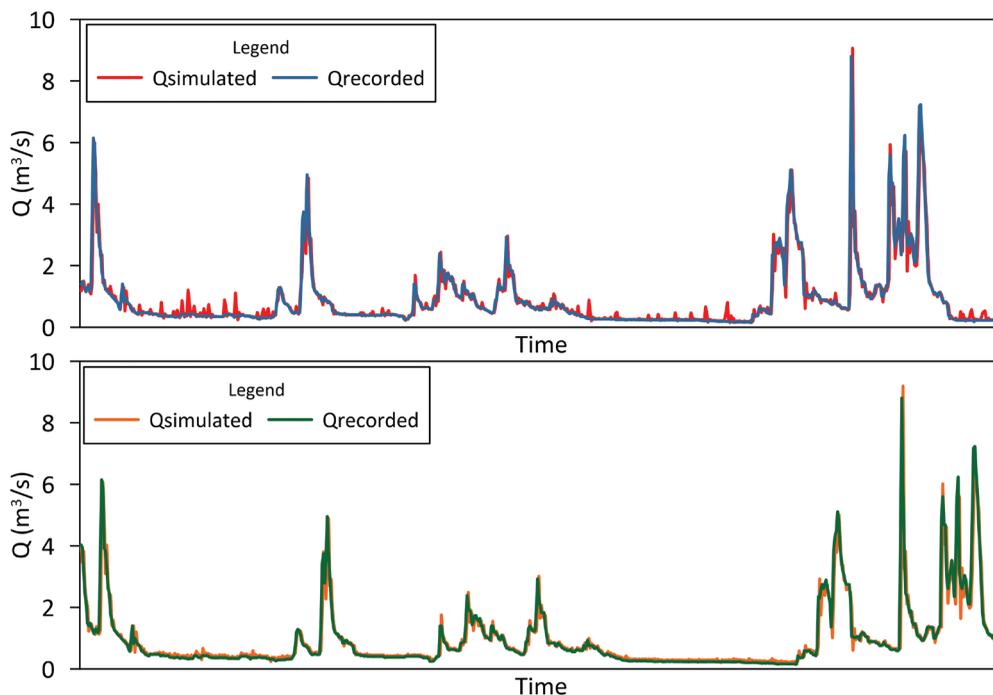


Fig. 1. Comparative diagram of simulated and recorded discharge data of the Seljašnica karst spring by applying simple ARCR model (above) and ARCR with linear moving average filter (below)

shows better matching of simulated and monitored discharge data not only in the aquifer recharge period, but also in the recession period.

Conclusion

Forecasting of Seljašnica karst spring discharge was carried out by applying stochastic modeling techniques. Autocorrelation and cross-correlation function, AR, CR and combined ARCR model were applied in order to properly simulate the spring discharge rates. Considering the large differences between simulated and recorder discharge rates in recession period by using simple ARCR model, a linear moving average filter was applied on precipitation data in order to improve the simulation model. Transformed ARCR model gave much better results in the low water period, while the simulation results are almost same in the high and average water period. This simulation model may give relatively confident short-term forecast of the Seljašnica spring discharge rates.

Acknowledgments: Our gratitude goes to the Ministry of Education, Science and Technological Development of the Republic of Serbia for financing project “OI176022”.

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