A stochastic model for simulation of karst spring discharge: Case study – Seljašnica karst spring (SW Serbia)

Veljko Marinović, Branislav Petrović, Zoran Stevanović

Centre for Karst Hydrogeology, Department of Hydrogeology, Faculty of Mining and Geology, University of Belgrade 11000, Serbia; E-mail: veljko.marinovic@yahoo.com

Keywords: karst spring, stochastic modeling, ARCR model, Serbia.

Introduction

Karst rocks cover a significant part of the Earth’s surface. Considering that karst can accumulate large amount of groundwater of excellent quality, karst aquifers are one of the most significant sources of drinking water. It is estimated that between 20% and 25% of the world’s population uses karst groundwater. Among them is the population in Serbia as well, considering that almost 20% of total water supply in Serbia is from karst groundwater and it is estimated that Serbia has about 580 x 10^6 m^3 of karst groundwater as potential exploitation reserves (Stevanović, 1995). Even though it seems that this amount of water can meet all water demands, the most common problem with karst groundwater is its seasonal fluctuation of quality and quantity in the course of a hydrological year. For instance, heavy rains and floods can deteriorate the quality of karst groundwater (especially its turbidity and microbiology), while seasonal depletion of karst groundwater reserves can jeopardise the local water supply. Thus, it will be very useful if one can predict the behaviour of karst groundwater quality and quantity regime, no matter if it is short- or long-term prediction. Regarding groundwater quantity, the most important thing is to predict the extreme discharge rates – minimal and maximal values, which is particularly significant in recession period when the water demand are highest and spring discharge is in minimum. Unlike intergranular aquifers, where the creation of hydrodynamic models within certain software packages is common, this is not (yet) the case with karst aquifers. Taking into account the heterogeneity and anisotropy of karst rocks (i.e., limestone in this case), the applicability of deterministic models is minimized, since it is practically impossible to predict the spatial distribution of karst groundwater within the karst hydrogeological system. For this reason, stochastic models in some cases may be the better option for simulation of karst spring discharge rates rather than deterministic models. Stochastic modeling of karst spring discharge has been applied to the Seljašnica karst hydrogeological system. This karst system is part of the Babine karst plateau that belongs to the Dinarides. This karst plateau (and aquifer system) is located in SW Serbia at the very border with Montenegro (probably having a transboundary character) and is drained by three large karst springs: Seljašnica and Bučje in Serbia and Breznica in Montenegro. The plateau is a fluvial relict formed over the thick deposits of Triassic limestones, whose thickness varies from 150 to 750 m, exceptionally up to 1200 m, as a bedrock of the ophiolites of Jurassic age. Numerous karst features can be found, such as sinkholes, dry and blind valleys that are proof about the existence of the formerly well-developed river network. Some ponors and small caves exist as well. The Seljašnica karst aquifer was formed in the carbonate rocks of Triassic age. The recharge process of the karst aquifer is primarily dependent on precipitation regime (effective rainfall and melted snow). In bare karst areas, the karst aquifer is being recharged by the direct infiltration of atmospheric waters, while the aquifer can be recharged by the sinking of surface streams formed on ophiolites and young volcanic rocks (allogenic recharge) (Marinović, Petrović, 2018).
Methodology

As stated above, the ability to predict karst spring discharge rates can be very useful particularly in
the recession period of the year (i.e. in summer months). This can be achieved by applying stochastic
or numerical modelling techniques. Considering unpredictability of karst hydrogeological system
functioning, stochastic modelling is easier to apply
while at the same time the results are relatively
reliable. Stochastic modelling techniques that were
applied in the case of Seljašnica karst spring are
autocorrelation function (ACF), cross-correlation
function (CCF) and autoregressive-crossregression
model (ARCR). This modelling techniques were
used to simulate karst spring discharge and calculate
a short-term prediction of discharge rates with
an appropriate confidence level. Autocorrelation
is the correlation between successive values of the
same variable (Kresic, 2007), which defines a mea-
sure of the internal correlation within a time series.
Cross-correlation function determines the inter-
dependence of two random variables, a dependent
and an independent one. Cross-correlation analysis
gives better insight into the functioning of the karst
hydrogeological system, i.e. on the discharge re-
gime and the influence of the rainfall on the system.
Autoregression (AR), crossregression (CR) and
combined autoregressive-cross-regression model
(ARCR) models were applied to simulate the dis-
charge of the Seljašnica karst spring. Autoregres-
sive models are used to generate synthetic arrays
of independent time series, and in some cases, they
can be used to produce short-term (1–2 days) pre-
dictions (Krešić, 1991). Besides the AR model, cross-regression (CR) models can also be used
for simulation, where, besides the dependent vari-
able (spring discharge), the independently variable
(precipitation) for 1, 2, ..., n the previous days is
introduced. For simulation and short-term forecasts
of karst groundwater discharge, the best results are
obtained by the combined (ARCR) model, which
includes the AR and CR model. ARCR models
belong to the category of multivariate time series
models (Krešić, 2010) and include multiple linear
regression. In order to improve the reliability of
the model, precipitation data were transformed by using
a linear moving average filter (LMAF), since the
simple ARCR model includes only gross precipita-
tion, which particularly has an impact on the spring
discharge in the recession period.

Results

Stochastic modeling techniques have been applied
on time series of discharge and rainfall data. In
the case of Seljašnica karst spring, time series of
daily values of spring discharge as well as daily
precipitation values from the Sjenica rain gauge
in the period May 2016–May 2018 were consid-
ered. Autocorrelogram gives a preliminary insight
into the characteristics of the karst hydrogeologi-
cal system, i.e. the spring discharge characteristics
and its behavior during a defined time series. The
autocorrelation of the spring discharge data shows
that the system memory is about 40 days, after
which the random variable becomes independent.
However, peaks can be observed between days 40
and 58, as well as after day 85, which can be trig-
ergged by snowmelt and infiltration. On the other
hand, autocorrelation of precipitation data shows
a system memory of only 2 days, which is under-
standable taking into account the random nature
of this variable. The cross-correlogram shows the
reaction of large karst conduits with a delay of two
days, while the statistical influence of precipita-
tion on the discharge of Seljašnica karst springs
exists until the 25th day. Also, peaks are observed
around 18th day, which could generally be taken
as the reaction time of the entire catchment area to
the precipitation, or to the general porosity of this
aquifer. As for application of the AR and CR mod-
els, the coefficients of the autoregressive model
range from 0.922 for the model order 1 to 0.937
for the model order 10 which is the consequence
of the long-term memory. In contrast, the coeffi-
cients of the cross-regression model are low and
range from 0.03 for the model order 1 to 0.36 for
model order 10. The coefficients of the combined
autoregression-cross-regression (ARCR) model
vary in the range from 0.928311 for model order 1
to 0.942365 for the model order 10. It can be seen
that with increasing order of the model, the validity
of the model does not increase significantly, based
on the coefficients of the model. Therefore, Krešić
(2010) states that when the time series are highly
autocorrelated, the order of model 1 or 2 (excep-
tionally up to 4) gives quite satisfactory results
in hydrogeological practice, which was used for
the simulation of the Seljašnica spring discharge
(Fig. 1). Fig. 1 shows that high and average wa-
ter periods are generally well simulated, while the
highest oscillations occur during low water peri-
ods. Such a result is due to the fact that this model
takes gross precipitation, which affects the large
differences in measured and simulated values dur-
ing the recession period. A linear moving average
filter was applied to avoid these problems. This
filter transforms the gross precipitation based on a
predetermined moving average window, which is
expressed in days. A moving average window of
40 days was applied to the existing ARCR model,
which generally corresponds to the memory of the
karst hydrogeological system calculated using the
autocorrelation function. Thus, Figure 1 clearly
shows better matching of simulated and monitored discharge data not only in the aquifer recharge period, but also in the recession period.

**Conclusion**

Forecasting of Seljašnica karst spring discharge was carried out by applying stochastic modeling techniques. Autocorrelation and cross-correlation function, AR, CR and combined ARCR model were applied in order to properly simulate the spring discharge rates. Considering the large differences between simulated and recorder discharge rates in recession period by using simple ARCR model, a linear moving average filter was applied on precipitation data in order to improve the simulation model. Transformed ARCR model gave much better results in the low water period, while the simulation results are almost same in the high and average water period. This simulation model may give relatively confident short-term forecast of the Seljašnica spring discharge rates.

**Acknowledgments:** Our gratitude goes to the Ministry of Education, Science and Technological Development of the Republic of Serbia for financing project “OI176022”.

**References**


