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## Stochastic simulation and prediction of turbidity dynamics in karst systems. Case study: Mokra karst spring (SE Serbia)

### Стохастична симулация и прогнозиране на динамиката на мътността в карстови системи по примера на карстов извор Мокра (ЮИ Сърбия)

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**Abstract.** Characterization of a karst system includes the analysis of two components – quantitative and qualitative one. Forecasting of future values of groundwater parameters can be very useful in defining the amounts of water needed for a reliable water supply. Stochastic simulation and forecasting were carried out for time series of precipitation and Mokra karst spring turbidity recorded in 2015. Simulation models within groundwater management would have a function in the early warning system which will enable timely response of groundwater source management.

**Keywords:** karst spring, stochastic modeling, ARCR model, Serbia.

### Introduction

Karst groundwater is a vital resource worldwide, primarily as a resource of drinking water for the population. It is estimated that about 2 billion people in the world use groundwater as a drinking resource, out of which almost 700 million use karst groundwater (Stevanović, 2019). Karst systems are hydrogeological environments formed in carbonate or evaporitic rocks, capable of accumulating significant amounts of high-quality groundwater. Characterization of a karst system includes the analysis of two components – quantitative and qualitative one. While the quantitative component refers to the study of flow and hydrodynamic properties of karst systems, the qualitative component of karst systems includes analysis of physicochemical, microbiological, gaseous and bacteriological properties of groundwater, variability of concentration of groundwater chemical parameters over time and attenuation capacity of a karst system. Successful characterization demands monitoring of karst groundwater continuously. Monitoring of karst groundwater has a very important role in defining the strategy of groundwater resource exploitation in

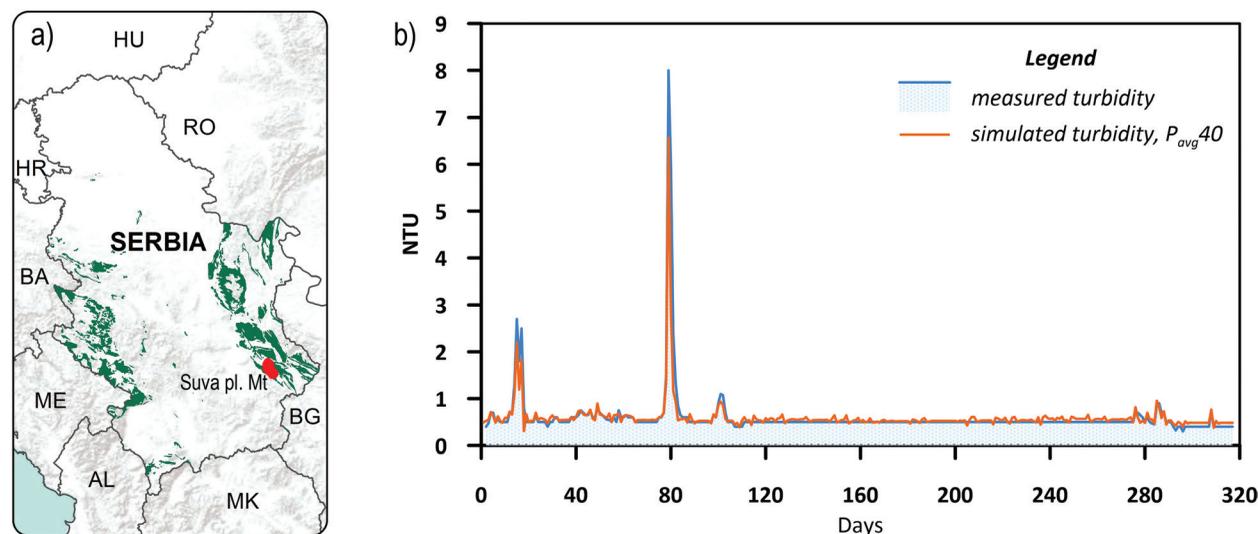
a rational and sustainable way. The collected data could provide material for prediction of quality or quantity changes of karst groundwater in the (near) future. Forecasting of future values of groundwater quality parameters (e.g. turbidity or electrical conductivity values) can be very useful in defining the amounts of water needed for a reliable water supply, as it is known that certain chemical parameters directly depend on quantitative characteristics of karst groundwater. Therefore, this paper deals with the applicability of stochastic time series analysis and modeling for simulation of certain qualitative parameters of karst groundwater, which can predict karst groundwater quality. Such a thing is common in hydrogeological practise (Margeta, Fistanic, 2004; Kovačić, 2010; Kovačić, Ravbar, 2015; Ristić Vakanjac et al., 2018). The stochastic model was created for the Mokra karst spring, located in the municipality of Bela Palanka, at the eastern foothill of Suva Planina Mt, SE Serbia (Fig. 1a). The geological structure of Suva Planina Mt is a consequence of multiple tectonic events, which led to the formation of the anticline of the NW-SE direction and the later uplift of its northwestern part (Vujsiće et al., 1971). The uplift of the anticline led to the erosion of the

overlying carbonate sediments and the exposure of the anticline core in the NW area, which is built by clastic Devonian and Permian sediments. Carbonate sediments, commonly limestones (and dolomites) of various degrees of purity, form the limbs of the anticline that dip towards the northeast and southwest. Karstification degree of the carbonate rocks of the Mesozoic age was significantly influenced by the tectonic pattern and limestone purity, so karst-fissure or karst aquifer types can be found (Petrović, Marinović, 2021). Recharge of karst aquifer is exclusively by infiltration of atmospheric precipitation. The main direction of groundwater movement within this karst massif is from the axis of the anticline towards the eastern and western wings, along the fault surfaces. Groundwater discharge takes place at the lowest points of the erosion base and Mokra karst spring is just one, and the strongest, of discharge points on the eastern foothill of mountain. Maximum yield values are over 4 m<sup>3</sup>/s, however during the summer months the yield decreases significantly to about 100 l/s.

## Methodology

In order to prevent possible occurrences of bacteriological and microbiological groundwater pollution, or to be able to predict the moment of occurrence of high concentrations of certain chemical parameters in groundwater, time series analysis and stochastic simulation are applied. Bearing in mind the great inhomogeneity of karst systems, stochastic analysis methods have a greater application in this area than the creation of numerical and deterministic models. Stochastic analysis involves the examination and description of specific time series using statistical

parameters. The main characteristic of the stochastic model is its definiteness with the probability of occurrence in time  $t$  based on the knowledge of the previous state in time  $t_0 < t$ . Time series analysis includes using auto- and cross-correlation functions, while the simulation can be performed using some of the stochastic models, most often autoregressive, cross-regressive and combined autoregressive-cross-regressive models. These models are used to simulate qualitative parameters in a defined period with a certain confidence interval. The autocorrelation function involves describing successive members of the same time series (i.e. dependent or independent variables) and determining its dependence and periodicity (Krešić, 1991; Larocque et al., 1998; Jemcov, 2008). This function expresses the correlation structure of the time series between its two mutual series, shifted by a certain time lag. The cross-correlation function implies determining the interdependence of two discretized variables, one of which is dependent and the other independent variable. Analysis the time series of karst groundwater quality means the examination of correlation of input signal to the system (e.g. precipitation) and turbidity, electrical conductivity or some other chemical parameter observed at the source (output signal from the system). In this paper, the influence of precipitation on turbidity values is considered, i.e. the time required the karst system to react to the recharge. Autoregressive (AR) models are used to generate synthetic sequences of independent time series, and in some cases can also be used to issue short-term (1–2 days) forecasts (Krešić, 1991). Using these models, a linear regression dependence is established between the members of the sequence of the same time series. Cross-regression (CR) models



**Fig. 1.** a, case study location – red, karst areas in Serbia – green; b, comparative diagram of measured and simulated turbidity values of Mokra karst spring by applying ARCR model with linear moving average filter of 40 days

are used to generate synthetic time series of a certain qualitative parameter as an output signal based on precipitation as input signal. For simulation and short-term forecasts of qualitative parameters of karst groundwater, the best results can be obtained with a combined autoregressive-cross-regressive (ARCR) model, which includes AR and CR models. ARCR models belong to the category of multivariate time series models (Krešić, 2010) and include multiple linear regression. To improve the reliability of the model, certain transfer functions (usually moving average) can be used in order to linearize and transform the input data (usually precipitation), in order to obtain more reliable inputs of the model.

## Results

Stochastic modelling techniques have been applied on time series of turbidity and rainfall data. Time series of daily values of turbidity measured *in situ* at Mokra karst spring as well as daily precipitation values from the Niš meteorological station in the year 2015 were considered. Simulation and short-term forecast of turbidity of groundwater of Mokra karst springs was performed using a combined autoregressive-cross-regressive (AR-CR) model, while gross precipitation was transformed using a linear moving average (MA) filter. A simulation was performed with several different moving average windows, but these simulations didn't show improved results. Finally, a 40-day moving average window was accepted and applied to the AR-CR model (Fig. 1b), which generally corresponds to the memory of a karst hydrogeological system calculated using autocorrelation function and cross-correlation analysis. Diagram (Fig. 1b) shows that maximum turbidity values are generally well simulated, while in simulation of minimal values of turbidity small oscillations occur. This result was obtained since this model takes the total gross precipitation, which greatly affects the large differences in measured and simulated values during the low water period (recession) when turbidity is usually very low (<0.1 NTU).

## Conclusion

Combined ARCR models were applied to properly simulate and forecast turbidity dynamics in Mokra karst aquifer. Stochastic simulation was carried out for time series of precipitation and turbidity recorded in 2015. Considering high residual values in summer months by using simple ARCR model, linear moving average filters were applied on precipitation data to improve the simulation model. Transformed ARCR model gave much better results, which is also validated by coefficient of correlation of simulated and measured turbidity values

( $R=0.95$ ) and ANOVA table. This simulation model may give relatively confident short-term forecast of Mokra karst spring turbidity values. In this way, simulation models within groundwater management would have a function in the early warning system, if reliable models would announce a possible deterioration of groundwater quality and/or quantity a few days in advance, which will enable timely response of the authorities. In that way, the closure of GW sources in periods when the water quality is highly deteriorated may be prevented.

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